ON METRIC PROPERTIES OF SETS OF ANGULAR LIMITS OF MEROMORPHIC FUNCTIONS

J. E. MCMILLAN*

Let f be a nonconstant function meromorphic in the unit disc $D=\{|z|<1\}$, with circumference C, and let E_z be a subset of C with positive (linear) measure. Suppose that at each $\zeta \in E_z$, f has an angular limit a_ζ, and let $E_w=\{a_\zeta : \zeta \in E_z\}$. It is known that E_w contains a closed set with positive harmonic measure (see Priwalow [6, p. 210] or Tsuji [7, p. 339]). Also known is that even when f is a schlicht function mapping D onto the interior of a Jordan curve, it may happen that E_w has linear measure zero (see Lavrentieff [2]) ; and a recent theorem of Matsumoto [4, p. 133] states, in effect, that if f is a schlicht function mapping D onto the interior of a Jordan curve, then E_w cannot have $\frac{1}{2}$-dimensional measure zero (For the definitions of (exterior) linear measure and α-dimensional measure zero ($\alpha>0$), see [5, pp. 149, 150].). The purpose of the present paper is to prove a theorem that generalizes Matsumoto's theorem. As a corollary of our theorem, we obtain: *If each point of E_w is accessible (with a Jordan arc) through the complement of $f(D)=\{f(z) : z \in D\}$, then E_w contains a closed set that does not have $\frac{1}{2}$-dimensional measure zero.*

If E_w is all of the extended w-plane Ω, the desired conclusion already holds ; so that we may, by first subjecting Ω to a linear transformation, *assume that* $\infty \notin E_w$. Our result is most conveniently expressed in terms of the Riemann surface S of f over Ω. For each $\zeta \in E_z$ and positive number h, let $S(\zeta, h)$ be the component of S over $\{|w-a_\zeta|<h\}$ such that if r is sufficiently near 1 ($r<1$), then $r\zeta$ corresponds under f to a point of $S(\zeta, h)$; and let $PS(\zeta, h)$ be the projection of $S(\zeta, h)$ onto Ω.

We prove

THEOREM. *Suppose that to each $\zeta \in E_z$ there correspond a Jordan arc γ_ζ (contained in the finite w-plane) with one endpoint a_ζ and a positive number h_ζ such*

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