ON THE GROTHENDIECK RING OF AN ABELIAN p-GOUP

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Introduction

The Grothendieck ring of a finite group has been studied by Swan ([5], [6]). At the end of [6] he determined completely the structure of the Grothendieck ring $G(Z\mathfrak{G})$ of a cyclic p-group $\mathfrak{G}$ over the ring of rational integers Z.

In this paper we investigate the structure of $G(Z\mathfrak{G})$ of an abelian p-group $\mathfrak{G}$.

In the first section we consider some properties of the integral group ring of $\mathfrak{G}$. The results of this section are applied in the second section to investigate the additive structure of $G(Z\mathfrak{G})$. Let $\mathfrak{o}$ be a maximal order of the group ring $Q\mathfrak{G}$ over the rational number field Q and let $Co(\mathfrak{o})$ be the reduced projective class group of $\mathfrak{o}$ (Rim [4]). We show that $G(Z\mathfrak{G})$ is isomorphic to the splitting Z-algebra extension of $Co(\mathfrak{o})$ by $G(Q\mathfrak{G})$ (§2, §3). The latter half of the third section is devoted to study the action of $G(Q\mathfrak{G})$ to $Co(\mathfrak{o})$. Some examples are given in the final section.

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§1. The integral group ring of a finite abelian group

Let R be the ring of integers of an algebraic number field K. The group ring $K\mathfrak{G}$ of a finite abelian group $\mathfrak{G}$ over K decomposes into a direct sum of algebraic number fields K_i over K

$$K\mathfrak{G} = K_1 \oplus \cdots \oplus K_s, \tag{1.1}$$

and $K_1, \ldots, K_s$ are a full set of non-isomorphic irreducible $K\mathfrak{G}$-modules. This decomposition induces the decomposition of the maximal order $\mathfrak{o}$ of $K\mathfrak{G}$ into a direct sum of maximal orders $\mathfrak{o}_i$ of K_i, i.e. the ring of integers of K_i. Since

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