RELATIVE COHOMOLOGY OF ALGEBRAIC LINEAR GROUPS, II

HIROSHI KIMURA

1. Introduction

Let G be an algebraic linear group over a field F of characteristic 0, and let H be an algebraic subgroup of G. Let A, M be rational G-modules. In [4], we defined $\mathrm{Ext}^n_{(G,H)}(A, M)$, and, in particular, relative cohomology groups $H^n(G, H, M)$ were defined as $\mathrm{Ext}^n_{(G,H)}(F, M)$.

$\mathrm{Ext}^1_{(G,H)}(A, M)$ may be identified with the space of the equivalence classes of the rational (G, H)-extensions of M by A ([4]). Moreover $\mathrm{Ext}^n_{(G,H)}(A, M)$ may be identified with the set of the equivalence classes of the rational n-fold (G, H)-extensions of M by A (Th. 2.2).

Let G be a unipotent algebraic linear group. Then there exists the natural homomorphism of $H^n(G, H, M)$ into the Lie algebra cohomology group $H^n(\mathfrak{g}, \mathfrak{h}, M)$, where $\mathfrak{g}$, $\mathfrak{h}$ are Lie algebras of G, H respectively. In Section 3, we show that, if M is finite dimensional, then the natural homomorphism $H^2(G, H, M) \to H^2(\mathfrak{g}, \mathfrak{h}, M)$ is surjective.

G. Hochschild studied the properties of rational injective modules ([3]). In Section 4, we obtain analogous results as described in [3].

2. Extensions of rational modules

Let G be an algebraic linear group over a field F, and let H be an algebraic subgroup of G. We denote by $R(G)$, or simply by R, the F-algebra of rational representative functions on G. If $f \in R$ and $x \in G$, the left and right translations, $x \cdot f$ and $f \cdot x$ of f by x are defined by $(x \cdot f)(y) = f(yx)$, $(f \cdot x)(y) = f(xy)$ for all $y \in G$. Let M be a rational G-module in the sense of [2]. We make the tensor product $R \otimes M$ over F into a G-module such that $x(f \otimes m) = f \cdot x^{-1} \otimes x \cdot m$. Then $R \otimes M$ is a rational G-module. We denote by ${}^H R$ the set consisting of the elements left fixed by left translations from H. Then ${}^H R \otimes M$ is a rationally (G, H)-injective submodule of $R \otimes M$ in the sense of [4] ([4, Prop. 2.1]).

Received March 22, 1965.