HOMOLOGY OF NON-COMMUTATIVE POLYNOMIAL RINGS

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§ 1. Introduction

Let Γ be a ring with unit element and let Λ be the Ore extension of Γ with respect to a derivation d of Γ [4, 3]. It is shown in [3] that l.gl. dim $\Lambda \leqq 1 +$ l.gl. dim Γ. It is not in general possible to replace this inequality by equality.

We consider here the special case where Γ is the polynomial ring in n variables over a commutative ring K. If d is a K-derivation of Γ then Λ becomes a K-algebra and we prove that if further Λ is a supplemented K-algebra, we have l.gl.dim $\Lambda = 1 +$ l.gl.dim Γ (Theorem 1). The proof consists first in constructing a Λ-free complex of length $n+1$ for K, which we prove to be acyclic (Proposition 2) by putting a suitable filtration on this complex and passing to the associated graded. We use this resolution to prove that $\text{l.dim}_\Lambda K = n+1$. We then employ a spectral sequence argument to complete the proof of Theorem 1. If Λ is not supplemented, Theorem 1 is not necessarily valid [5].

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§ 2

Let K be a commutative ring with 1 and let $\Gamma = K[x_1, \ldots, x_n]$ be the polynomial ring in n variables over K. Let d be a K-derivation of Γ into itself. Clearly d is uniquely determined by its values f_i on x_i. Conversely, given n polynomials $f_i \in \Gamma$, $1 \leq i \leq n$, there exists a K-derivation d of Γ into itself with $d(x_i) = f_i$, $1 \leq i \leq n$.

Let Λ be the non-commutative polynomial ring in one variable x_{n+1} over Γ with respect to d. Then Λ is the K-algebra with generators $x_1, \ldots, x_{n+1}$ and relations given by

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