ON GROUPS WITH NILPOTENT COMMUTATOR SUBGROUPS

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§ 1. Introduction

It is well known that a supersoluble group has the nilpotent commutator subgroup. But the converse does not hold in general. Therefore, it will be interesting to study the class of groups with nilpotent commutator subgroups. Some results were obtained by B. Huppert [1] and R. Baer [3] on this subject.

It is easily seen that the p-length of a group with the nilpotent commutator subgroup is 1 for every p dividing the order of this group. Theorem 1 of this paper may be considered as its local refinement, and is proved in a similar way as B. Huppert [2]. Theorem 2 gives a necessary and sufficient condition for a group to have the nilpotent commutator subgroup in terms of its Hall subgroups. In the last theorem 3 we shall study an interesting class of groups which have maximal subgroups of a certain nature. It turns out that these groups have nilpotent commutator subgroups and admit Sylowtowers.

The auther wishes to express his heartfelt thanks to A. Hattori for his kind leading.

§ 2. Notations and definitions

$\vert G\vert$	order of a finite group G.
G'	commutator subgroup of G.
G''	commutator subgroup of G'.
$H\subset G$	H is a subgroup of G.
$N_G(H)$	normalizer of H in G.
$C_G(H)$	centralizer of H in G.
$\Phi(G)$	Frattini subgroup of $G=$ the intersection of all maximal subgroups of G.
$\Delta(G)$	the intersection of all maximal subgroups of G which are not normal in G.

Received August 18, 1964.