ON THE DISTORTION AND CORRESPONDENCE UNDER QUASICONFORMAL MAPPINGS IN SPACE

KAZUO IKOMA

Introduction

Recently many important results on rings and quasiconformal mappings in space have been obtained by B. V. Šabat [9], F. W. Gehring [3], J. Väisälä [11] and others. The modulus of a ring in space can be defined in three apparently different but essentially equivalent ways. (See Gehring [4]). In the theory of quasiconformal mappings in space, some properties for moduli of rings in space play an important role, because the method by means of moduli acts also as a substitute in space for the Riemann mapping theorem which does not hold in space.

In this paper, we shall give the extremal condition for spherical rings, an extension of Grötzsch-Šabat's theorem and an example of space K-quasiconformal mappings constructed by associating with certain plane K-quasiconformal ones. By using these results, the monotoneity for moduli of space rings (B. Fuglede [1]) and the extremality and estimates for moduli of the space Grötzsch and Teichmüller rings (Gehring [2]) as our main tools, we shall establish mainly the space forms corresponding to our previous results ([5], [6]) of plane K-quasiconformal mappings. That is to say, we are concerned with the Schwarz's lemma for space K-quasiconformal mappings, some distortions in these mappings (under a certain normalization at the origin) and a criterion for two sets corresponding to each other by such mappings to be of the same dimensional Hausdorff measure zero, where it is remarked that even if one of such mappings transforms a plane domain into another plane domain, its correspondence between these plane domains does not always induce a plane K-quasiconformal mapping.

For the sake of convenience, we shall mostly restrict rings and K-quasiconformal mappings to the 3-dimensional space.

The author wishes to express his hearty gratitude to Professors K. Noshiro

Received June 1, 1964.