CROSSED PRODUCTS AND MAXIMAL ORDERS

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Introduction. Let Γ be a maximal order over a complete discrete rank one valuation ring R in a central simple algebra over the quotient field of R. The purpose of this paper is to determine necessary and sufficient conditions for Γ to be equivalent to a crossed product over a tamely ramified extension of R.

It is a classical result that every central simple algebra over a field k is equivalent to a crossed product over a Galois extension of k. Furthermore, it has been proved by Auslander and Goldman in [2] that every central separable algebra over a local ring is equivalent to a crossed product over an unramified extension.

Let R denote a discrete rank one valuation ring. The set of maximal orders $M'(R)$ over R forms a subset of the set of hereditary orders $H'(R)$ over R (see [3]). An equivalence relation on the set of hereditary orders has been defined in [2]. Namely, if Λ_1 and Λ_2 are in $H'(R)$, then Λ_1 is said to be equivalent to Λ_2 if there exist finitely generated free R-modules E_1 and E_2 and an R-algebra isomorphism

$$\Lambda_1 \otimes_R \mathrm{Hom}_R(E_1, E_1) \cong \Lambda_2 \otimes_R \mathrm{Hom}_R(E_2, E_2).$$

It is established in [2] that an hereditary order which is equivalent to a maximal order is itself a maximal order.

The author has proved in [10] that every crossed product $\Delta(f, S, G)$ over a tamely ramified extension S of a discrete rank one valuation ring R is an hereditary order, and that $\Delta(f, S, G)$ is a maximal order if and only if the order of the conductor group H_f is one (see Section 1 for the definition of H_f). She has also exhibited in this paper an example of a non-maximal hereditary order which is not equivalent to a crossed product over a tamely ramified extension. Now let Γ be a maximal order over a complete discrete rank one valuation ring R in a central simple algebra Σ over the quotient field of R.

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