ON CERTAIN MAPPINGS OF RIEMANNIAN MANIFOLDS

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In this paper we consider certain tensors associated with differentiable mappings of Riemannian manifolds and apply the results to a p-mapping, which is a special case of a subprojective one in affinely connected manifolds (cf. [1], [7]). The p-mapping in Riemannian manifolds is a generalization of a conformal mapping and a projective one. From a point of view of differential geometry an analogy between these mappings is well known. On the other hand it is interesting that a stereographic projection of a sphere onto a plane is conformal, while a central projection is projectve, namely geodesic-preserving. This situation was clarified partly in [6]. A p-mapping defined in this paper gives a precise explanation of this and also affords a certain mapping in the euclidean space which includes a similar mapping and an inversion as special cases.

1. Tensors associated with mappings of Riemannian manifolds

1. Let M and N be two n-dimensional Riemannian manifolds of differentiable class C^3 and φ be a mapping of differentiable class C^3 which is locally regular. We take a coordinate neighborhood U and differentiable sets of orthogonal coframes on U and $\varphi(U)$. Then the arc-elements of M and N are given respectively as

$$ds^2 = \sigma^i\sigma^i, \qquad dt^2 = \tau^i\tau^i. \tag{1.1}$$

Now we put

$$\varphi^*\tau^i = p^i_j\sigma^j, \qquad a_{ij} = p^k_i p^k_j. \tag{1.2}$$

Then we have $det(a_{ij}) \neq 0$ on account of regularity and

$$\varphi^* dt^2 = \varphi^*(\tau^i\tau^i) = a_{ij}\sigma^i\sigma^j, \qquad (a_{ij} = a_{ji}). \tag{1.3}$$

We call $A = (a_{ij})$ the first tensor of our mapping φ.

Next we take forms of Riemannian connections

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