CLASSIFICATION OF LOCALLY EUCLIDEAN SPACES

LEO SARIO

1. The classification of Riemann surfaces has largely reached its completion. The purpose of the present paper is to lay the foundation for a new intriguing field in the classification theory: Riemannian spaces with Euclidean metrics. The paper is self-contained, both for the Riemann surface expert and the reader whose main interest is with higher dimensions.

The significance of locally Euclidean spaces lies, first of all, in that their function-theoretic nature differs for dimensions $n>2$ and $n=2$. The existence or nonexistence of Green's functions and positive or bounded harmonic functions in R^n, punctured R^n, and in the punctured flat torus offer simple examples. A striking phenomenon is that, despite such differences, the basic inclusion relations remain valid. Moreover, capacities and null-classes can be defined for components of point sets in R^n.

These results are established by an extension of the linear operator method ([6], [7]). The main points of the generalized method are given in Nos. 2, 3, 8, 17, 21, and 23-27. The significance of this extension is in the fact that the absence of such devices as conformal mappings, conjugate harmonic functions, the reflection principle, and doubling of bordered regions necessitates new tools.

Another promising aspect of higher dimensions is the introduction of new function classes (Nos. 29-34). In No.35 we give a list of questions, an essential part of our paper. The important unsolved problem on the strictness of the inclusion $O_{HB} \subset O_{HD}$, well-known for Riemann surfaces (No. 24), is typical of these.

Several interesting topics are meaningful only in locally Euclidean spaces. However, at the cost of somewhat heavier equipment, some of our reasoning can be generalized to arbitrary Riemannian spaces.

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