A CERTAIN KIND OF FORMAL THEORIES

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Introduction

A common feature of formal theories is that each theory has its own system of axioms described in terms of some symbols for its primitive notions together with logical symbols. Each of these theories is developed by deduction from its axiom system in a certain logical system which is usually the classical logic of the first order.

There are many formal theories of mathematics, e.g. the natural number theory, the Euclidean geometry, the set theory, etc. Naturally, a single theory can be formulated in various ways just as we can describe a theory in various languages. There are, however, many mathematical theories which seem essentially different to each other; for instance, the natural number theory seems quite different from the Euclidean geometry. On the other hand, these formal theories are linked together closely by some basic theory, usually the set theory. The linkage is exhibited in reducing consistency of each theory meta-theoretically to consistency of the basic theory. We can really confirm meta-theoretically consistency of the natural number theory as well as the Euclidean geometry by assuming that the set theory is consistent. In the course of such meta-theoretical reasoning, the dominion of the basic theory is enlarged step by step. As a matter of course, we have been seeking for a consistent basic system as simple and as dominant as possible, and we have composed our trial systems **OZ** and **OF** along this line. (See Ono [9] and [10].) Anyway, our chief concern has been the axiom systems of formal theories, not their logic. The logic of the basic theory has been supposed to remain unchanged.

However, is it really our natural way of free thinking to develop special theories in a certain basic theory without bringing up logic itself? In the primitive stage, our logic would concern with real objects only; while in the developed stage, our logic must concern with abstract objects in the world of possibility. Are our logics quite the same for real objects and for abstract objects? Even

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