COMPACTIFICATIONS OF HARMONIC SPACES

C. Constantinescu and A. Cornea

Many results of the theory of Riemann surfaces derive only from the properties of the sheaf of harmonic functions on these surfaces. It is therefore natural to try to extend these results to more comprehensive structures defined by means of a sheaf of continuous functions on a topological space which should possess the main properties of the sheaf of harmonic functions on a Riemann surface. The aim of the present paper is to generalise some known results from the theory of Riemann surfaces to spaces endowed with sheaves satisfying Brelot's axioms [2], which we call harmonic spaces. In order to do so we had to introduce and to study the maps, associated in a natural way with this structure, called harmonic maps; they replace the analytic maps between Riemann surfaces. In this general frame we reconstruct the whole theory of Wiener compactification as well as the theory of the behaviour of analytic maps at the Wiener boundary.

The first paragraph contains some simple remarks about the Dirichlet problem which could not be found in the existing literature. In the second paragraph we introduce and study the operator h and the Wiener functions which represent the main tool of the present paper. The harmonic maps are studied in the third paragraph. In the fourth paragraph we treat some problems concerning general compactifications and in the fifth one the particular case of Wiener compactification is considered. This compactification is closely related to Feller's ideal boundary. The last paragraph is devoted to the problem of the behaviour of the harmonic maps at the Wiener boundary.

Without mentioning the source we have borrowed intensively ideas, methods and usual tricks from various papers on the theory of Riemann surfaces. For many of them we are indebted to K. Hayashi, M. Heins, Y. Kusunoki, S. Mori, M. Nakai. A detailed bibliography in this direction may be found in our book "Ideale Ränder Riemannscher Flächen", Springer Verlag, 1963.

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