RADIAL LIMITS OF QUASICONFORMAL FUNCTIONS

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Beurling and Ahlfors [1] answered a fundamental question concerning the boundary correspondence induced by a quasiconformal mapping when they proved that the correspondence need not be given by an absolutely continuous function. They proved this by characterizing the boundary correspondences of quasiconformal mappings of the upper half-plane $Im(z)>0$ onto the upper half-plane $Im(w)>0$ under which the boundary points at infinity correspond. They proved that a necessary and sufficient condition that the strictly monotone increasing function $\mu(x)$ carrying the real axis onto itself be the boundary correspondence induced by such a quasiconformal mapping is that $\mu(x)$ should satisfy a ρ-condition

$$\frac{1}{\rho}\leqq\left[\frac{\mu(x+t)-\mu(x)}{\mu(x)-\mu(x-t)}\right]\leqq\rho \tag{1}$$

for some constant ρ and all real x and t. In addition they showed that if the ρ-condition is fulfilled there exists a quasiconformal mapping whose dilatation does not exceed ρ^2 and every mapping with the boundary correspondence $\mu(x)$ must have a maximal dilatation greater than or equal to $1+A\log\rho$ where A is a certain numerical constant ($=.2284$). For a given mapping $\mu(x)$ of the real axis denote by $\rho(\mu)$ the smallest value of ρ such that the ρ-condition (1) is fulfilled. Beurling and Ahlfors showed the stronger result that there exists a quasiconformal mapping of the half-plane onto itself whose boundary correspondence is given by a completely singular function $\mu(x)$ with $\rho(\mu)$ arbitrarily close to one. Because of this result and its function-theoretic consequence that the distinction between sets of zero and positive harmonic measure is not preserved under quasiconformal mappings, it follows that the analogue of Fatou's theorem does not hold for quasiconformal functions. The purpose of this note

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