REPRESENTATIONS OF ALGEBRAIC GROUPS

ROBERT STEINBERG*

To Professor RICHARD BRAUER on the occasion of his 60th birthday

§ 1. Introduction

Our purpose here is to study the irreducible representations of semisimple algebraic groups of characteristic $p \neq 0$, in particular the rational representations, and to determine all of the representations of corresponding finite simple groups. (Each algebraic group is assumed to be defined over a universal field which is algebraically closed and of infinite degree of transcendence over the prime field, and all of its representations are assumed to take place on vector spaces over this field.)

To state our first principal result, we observe that relative to a Cartan decomposition of a semisimple algebraic group, there is described in § 5 below (in a somewhat more general context) a standard way of converting an isomorphism on the universal field into one on the group, and that relative to a choice of a set S of simple roots, an irreducible rational projective representation of the group is characterized by a function from S to the nonnegative integers, to be called, together with the corresponding function on the Cartan subgroup of the decomposition, the high weight of the representation [13, Exp. 14 and 15].

1.1 THEOREM. *Let G be a semisimple algebraic group of characteristic $p \neq 0$ and rank l, and let $\mathfrak{R}$ denote the set of p^l irreducible rational projective representations of G in each of which the high weight λ satisfies $0 \leq \lambda(\boldsymbol{a}) \leq (p-1)$ $(\boldsymbol{a} \in S)$. Let α_i denote the automorphism $t \to t^{p^i}$ of the universal field as well as the corresponding automorphism (see § 5) of G, and for $R \in \mathfrak{R}$ let R^{α_i} denote the composition of α_i and R. Then every irreducible rational projective representation of G can be written uniquely as $\Pi_{i=0}^{\infty} R_i^{\alpha_i}$ (weak tensor product, $R_i \in \mathfrak{R}$).*

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