ON THETA FUNCTIONS AND ABELIAN VARIETIES OVER VALUATION FIELDS OF RANK ONE

(II) THETA FUNCTIONS AND ABELIAN FUNCTIONS OF CHARACTERISTIC $p(>0)$

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To RICHARD BRAUER on his 60th Birthday

It may safely said that one of the most important problems in modern algebraic geometry is to elevate theory of abelian functions to the same level as theory of elliptic functions beyond the modern formulation of classical results. Being concerned in such a problem, we feel that one of the serious points is the lack of knowladge on the explicit expressions of abelian varieties and their law of compositions by means of their canonical systems of coordinates: Such expressions correspond to the cubic relation $\wp'^2 = 4\wp^3 - g_2\wp - g_3$ of Weierstrass' $\wp$-functions and their addition formulae in theory of elliptic functions.

In Part (I) we have introduced theta functions and abelian functions over fields of characteristic p with valuations of rank one,[1] and have shown that for each positive symmetric bimultiplicative function q valued in a valuation field of rank one there exists an abelian variety A_q such that A_q is embedded in a projective space by means of theta functions of some type with period (E, q).

In the present part (II) first we shall give the explicit addition formulae of the following abelian functions of characteristic $p(\geq 3)$

$$\{\Phi_{\mathfrak{g}_i}(u) = \vartheta_p[g_i,\ 0](q\,|\,u)/\vartheta_p[0,\ 0](q\,|\,u)\,|\,p\mathfrak{g}_i \in \mathfrak{M}\}$$

as an immediate consequence from the fact that $\{\vartheta_p[\mathfrak{g}_i,\ 0](q\,|\,u)\}$ form a base of theta functions of type $(p, 1)$ with period (E, q); the explicit addition formulae are comparatively simple, and they may be considered as the formulae

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[1] We shall freely use the notations and results in Part (I), [2].