ON THE CLIFFORD COLLINEATION, TRANSFORM AND SIMILARITY GROUPS (IV)

AN APPLICATION TO QUADRATIC FORMS

G. E. WALL

To RICHARD BRAUER on his 60th birthday

1. Introduction

E. S. Barnes and I recently[1] constructed a series of positive quadratic forms f_N in $N=2^n$ variables $(n=1, 2, \ldots)$ with relative minima of order $N^{\frac{1}{2}}$ for large N. I continue this investigation by determining the minimal vectors of f_N and showing that, for $N \neq 8$, its group of automorphs is the Clifford group[2] $\mathscr{C}\mathscr{T}_1^+(2^n)$ (§3). This suggests a generalization. Replacing $\mathscr{C}\mathscr{T}_1^+(2^n)$ by $\mathscr{C}\mathscr{T}(p^n)$, where p is an odd prime, I derive a new series of positive forms in $N=(p-1)p^n$ variables (§4). The relative minima are again of order $N^{\frac{1}{2}}$ (p fixed, $N \to \infty$), the "best" forms being those for $p=3,5$. All forms are eutactic though only those for $p=3,5$ are extreme.

The methods used here raise several questions. Firstly, the forms constructed have fairly big relative minima while the representations of the symplectic group $Sp(2n, p)$ associated with $\mathscr{C}\mathscr{T}(p^n)$ are of smallest possible degree (CGI, theorem 10). Are these two facts directly related? Secondly, it is natural to regard the lattice introduced in §4.2 as a commutative algebra. Is there a simple direct relation between this algebra and the automorph group $\mathscr{C}\mathscr{T}(p^n)$?

2. Preliminaries

The notation used in this paper is a compromise between that of EF and that of CGI, CGII. See in particular §2.1–2.3 below.

2.1. Vector spaces and groups over GF(p).

Throughout this paper, p stands for a fixed prime and n for a fixed natural

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[1] Cf. [1]. This paper is referred to as EF.

[2] Cf. [2], [3]. These papers are referred to as CGI, CGII.