

ON A PRACTICAL WAY OF DESCRIBING FORMAL DEDUCTIONS

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Introduction

Even though the logical structure of any formal deduction can be nicely expressed in a tree-form diagram, it is more practical to write it down in a series of propositions. In each step of inference, we usually deduce a proposition on basis of some foregoing propositions¹⁾. However, global aspects of mathematical theories show us that this is not always the case. For, in mathematical theories, theorems are usually stated before their proofs. In fact, also in proofs of theorems, it is often practical that we prove propositions after stating them. Accordingly, in our real way of thinking, we arrange propositions going back and forth in the logical order.

In formal deductions, some propositions are stated as temporary assumptions, definitions, or something like that. Furthermore, propositions such as "Take any object, say x , satisfying the conditions $\mathfrak{C}(x)$." nominate temporarily the variable x , having the assumption character. On the other hand, propositions such as "There is an object t satisfying the condition $\mathfrak{C}(t)$, so take any one of such objects and call it x ." also nominate temporarily the variable x , having the assertion character. In such cases, we are tempted to use the universal or existential quantifiers, though confusions are hardly avoidable without introducing some devices.

In Section (1), we introduce new symbols of the forms $\forall x!$ and $\exists x!$ called nominating quantifiers to meet the demand. Namely, by making use of these quantifiers, we denote the former example as " $\forall x! \mathfrak{C}(x)$ " and the latter one as " $\exists x! \mathfrak{C}(x)$ ".

In Section (2), we introduce a system of numbering of all the propositions in any formal deduction. We use Latin letters for numbering propositions in

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¹⁾ Such is the case for the most of logical systems, e.g. Gentzen's systems [1]. Kuroda [2] introduced a system, in which inferences are described in the inverse direction.