A NOTE ON CONFORMAL MAPPINGS OF CERTAIN RIEMANNIAN MANIFOLDS

MINORU KURITA

The contents of this note were reported at a meeting of the Japan Mathematical Society five years ago, but it was not printed. Prof. K. Yano advised me to do so and it was as follows.

1. We take n-dimensional compact orientable Riemannian manifolds V and $\overline{V}$, and denote their line elements by ds^2 and $d\bar{s}^2$ and their scalar curvatures by R and $\overline{R}$ respectively (Signs of the curvatures are taken in such a way that they are positive for the spheres). We consider a conformal homeomorphism f from V to $\overline{V}$ and put

$$f^*(d\bar{s}^2) = a^2 ds^2 \qquad (a > 0),$$

where f^* means a mapping of differential forms dual to f. We take a neighborhood of any point of V and orthogonal frames on it. Then ds^2 can be written as $ds^2 = \sum_i \omega_i^2$ with 1-forms ω_i $(i = 1, \ldots, n)$. We put as usual

$$d(\log a) = \sum_i b_i \omega_i, \qquad b^2 = \sum b_i^2,$$

$$b_{ij} = \nabla_j b_i - b_i b_j + \frac{1}{2} b^2 \delta_{ij},$$

where ∇ means a covariant differentiation with respect to the Riemannian metric on V. Then we get a wellknown formula

$$R - \overline{R}a^2 = 2(n-1)\sum_i b_{ii}, \tag{1}$$

where we write $\overline{R}$ briefly instead of $f^*\overline{R}$. We take a number s which shall be determined later and put

$$a^s d(\log a) = dc = \sum_i c_i \omega_i. \tag{2}$$

Then we have $c_i = b_i a^s$ and

Received October 19, 1961.