ON THE BOUNDARY BEHAVIOR OF HOLOMORPHIC FUNCTIONS IN THE UNIT DISK

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I. Introduction

1. Let $f(z)$ be a holomorphic function defined in the unit disk $|z|<1$, which we shall denote by D. Let Σ be a subset of D, whose closure has at least one point in common with C, the circumference of the unit disk. The set of all values a such that the equation $f(z)=a$ has infinitely many solutions in Σ is called the *range of $f(z)$ in Σ*, and is denoted by $R(f, \Sigma)$. Let τ be a point of C, and let $\{z_n\}$ be a sequence of points in D with the properties: $z_n = r_n\tau$, $0<r_n<1$, $\lim\limits_{n\to\infty} r_n = 1$. The non-Euclidean (hyperbolic) distance $\rho(z_n, z_{n+1})$ between two points z_n and z_{n+1} of the sequence is defined to be equal to

$$\frac{1}{2}\log\frac{1+u}{1-u}, \quad u = \left|\frac{z_n - z_{n+1}}{1-\bar{z}_n z_{n+1}}\right|$$

(cf. [3], Ch. II).

We shall abbreviate the expression "non-Euclidean" to *n-E*. For a discussion of the *n-E* geometrical matters involved in this paper, the reader is referred to [3].

Given a point τ on C, the set of all points z in D for which

$$-\frac{\pi}{2}<\alpha<\arg(1-\bar{\tau}z)<\beta<\frac{\pi}{2}, \quad |z-\tau|<\varepsilon,$$

where α and β are given angles and ε is so small that the boundary of the resulting set has only the point τ in common with C shall be called a *Stolz angle at τ*. If $\alpha=-\beta$, the resulting set is called a *symmetric Stolz angle with vertex τ and of opening 2β*, and will be denoted by $\Delta_{\tau,\beta}$.

It is the purpose of the present paper to study the boundary behavior of

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