A PROPERTY OF META-ABELIAN EXTENSIONS

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Let k be an algebraic number field of finite degree, $\mathbf{A}$ the maximal abelian extension over k, and M a meta-abelian field over k of finite degree, that is, M/k be a normal extension over k of finite degree with an abelian group as commutator group of its Galois group. Then $\mathbf{A}M$ is a kummerian extension over $\mathbf{A}$. If its kummerian generators are obtained from a subfield K of $\mathbf{A}$, namely if there exist elements $a_1, \ldots, a_t$ of K such that $\mathbf{A}M = \mathbf{A}(\sqrt[m_1]{a_1}, \ldots, \sqrt[m_t]{a_t})$, then we shall call M a *meta-abelian field over k attached to K*. If furthermore there exist $b_1, \ldots, b_s$ of K such that $\mathbf{A}M = \mathbf{A}(\sqrt[n_1]{b_1}, \ldots \sqrt[n_s]{b_s})$ and M contains all n_i-th roots of unity ($i = 1, \ldots, s$), then we shall call M a *K-meta-abelian field over k* and $b_1, \ldots, b_s$ *M-reduced elements* of K. For k-meta-abelian fields over k, we have in [2] the decomposition law of primes of k in M.[1)] The purpose of the present paper is to show that this decomposition law is effective also for meta-abelian fields over k attached to k, or more exactly these fields are already k-meta-abelian fields over k. We shall have a little more generally the following

THEOREM. *If M is a meta-abelian field over k attached to K, then MK is a K-meta-abelian field over k.*

In order to prove the theorem it is sufficient to observe the case where K is equal to k. Now let M be a meta-abelian field over k attached to k, $\mathbf{A}_0$ the largest abelian subfield of M, and M_i a cyclic subfield of M over $\mathbf{A}_0$ whose degree is a power of a prime l. Then there exists an element a_i of k such

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1) The symbol $\left[\frac{a}{\mathfrak{p}}\right]_n$ is not defined in [2] for the case $r=0$. Therefore to state the decomposion law it is necessry that $r \geqq 1$, namely k containes all l-th roots of unity. But if we define $\left[\frac{a}{\mathfrak{p}}\right]_n = 1$ or $=0$ according as $a^{(N\mathfrak{p}^{\rho}-1)} \equiv 1$ or $\not\equiv 1$ (mod. $\mathfrak{p}$), then, remarking that lemma 4 in [2] is also true for $r=0$, we have the decomposition law in M/k by means of this symbol also for the case $r=0$. Here $\left[\frac{a}{\mathfrak{p}}\right]_n \left[\frac{b}{\mathfrak{p}}\right]_n = \left[\frac{ab}{\mathfrak{p}}\right]_n$ does not hold when $\left[\frac{a}{\mathfrak{p}}\right]_n = 0$.