CAPACITABILITY OF ANALYTIC SETS

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Introduction

Let Ω be a locally compact separable metric space and let Φ be a positive symmetric kernel. Then the inner and outer capacities of subsets of Ω are defined by means of Φ-potentials of positive measures in the following manner. We define the capacity $c(K)$ of a compact set K in a certain manner by means of Φ-potentials. By this set function we define the inner and outer capacities of a subset X of Ω as follows:

$$\mathrm{cap}_i(X) = \sup c(K) \text{ for all compact } K \text{ contained in } X,$$
$$\mathrm{cap}_e(X) = \inf \mathrm{cap}_i(K) \text{ for all open } G \text{ containing } X.$$

A subset whose inner capacity coincides with its outer capacity is said to be capacitable. In this paper we discuss whether or not every analytic set is capacitable, where an analytic set is, by definition, the continuous image of a $K_{\sigma\delta}$ set in a compact space.

As for the classical capacities, for example, the α-capacities $(0 < \alpha \leqq 2)$ in the m-dimensional euclidean space R^m $(m \geqq 3)$, the problem of the capacitability was solved affirmatively by Choquet [3]. This result was extended by Aronszajn and Smith [1] as follows: every analytic set in R^m is capacitable with respect to the α-capacities for all α, $0 < \alpha < m$. Here the α-capacities are defined by the set function

$$c^{(\alpha)}(K) = \inf_{\mu \in \mathfrak{E}_K} \int U^\mu_\alpha \, d\mu, \tag{1}$$

where U^μ_α denotes the α-potential of a positive measure μ, that is,

$$U^\mu_\alpha(x) = \int \frac{1}{|x-y|^{m-\alpha}} \, d\mu(y), \qquad x \text{ and } y \in R^m,$$

and $\mathfrak{E}_K$ denotes the family of positive measures μ such that the α-potential $U^\mu_\alpha \geqq 1$ on K with a possible exception of a set E which is of measure zero with

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