ON THE RING OF INTEGERS IN AN ALGEBRAIC NUMBER FIELD AS A REPRESENTATION MODULE OF GALOIS GROUP

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1. Introduction. It is known that there are only three rationally inequivalent classes of indecomposable integral representations of a cyclic group of prime order l. The representations of these classes are:

(I) identical representation,

(II) rationally irreducible representation of degree $l-1$,

(III) indecomposable representation consisting of one identical representation and one rationally irreducible representation of degree $l-1$ (F. E. Diederichsen [1], I. Reiner [2]).

We now consider the special case where the representation module is the ring of algebraic integers of a number field and the operator group is a cyclic group of Galois automorphisms of prime order, and show that the multiplicity in this representation of indecomposable components belonging to each one of the above standing rationally inequivalent classes is determined by ramification numbers.[0)]

2. Theorem on the different. In this note, we denote by $\mathfrak{o}_\Omega$ for an algebraic number field Ω the ring of integers of Ω and by $D_{\Omega/L}$ for an extension Ω of an algebraic number field L the relative different of Ω/L.

The main aim of this article is to prove the following

THEOREM 1. *Let k be an algebraic number field of finite degree and K be a normal extension of k. Then the relative traces of all integers of K to k constitute an integral ideal of k and the ideal is characterized as the maximal divisor of k dividing the relative different $D_{K/k}$.*

We must first establish two lemmas.

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0) In the case of absolutely abelian number fields, some results in this note have recently been proved by H. W. Leopoldt [2a]. (This foot-not and [2a] are added September 15, 1959.)