SOME REMARKS ON SYMMETRIC AND FROBENIUS ALGEBRAS

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In [5] we defined the concepts of Frobenius and symmetric algebra for algebras of infinite vector space dimension over a field. It was shown there that with the introduction of a topology and the judicious use of the terms continuous and closed, many of the classical theorems of Nakayama [7, 8] on Frobenius and symmetric algebras could be generalized to the infinite dimensional case. In this paper we shall be concerned with showing certain algebras are (or are not) Frobenius or symmetric. In Section 3, we shall see that an algebra can be symmetric or Frobenius in "many ways". This is a problem which did not arise in the finite dimensional case.

In Section 1, we consider algebras of transformations in an infinite dimensional vector space. We show that the algebras of transformations of finite rank are symmetric, but that the algebra of all transformations is not even Frobenius. The latter statement is proved by means of a lemma which shows (among other things) that every transformation in an infinite dimensional vector space is actually a commutator.

In Section 2 we consider tensor products of Frobenius and symmetric algebras. *We also show that under certain conditions* the inner product on a Frobenius algebra can be normalized so that $(1, 1) = 1$.

In Section 3, we show that the polynomial ring with coefficients in a field can be made into a symmetric algebra in an uncountable number of ways. As a consequence of this, the polynomial ring has an uncountable number of inequivalent (metrizable) topologies such that it is a topological ring with respect to each and in every such topology every non zero ideal is dense in the whole algebra.

The term topological ring here requires, along with the usual assumptions about addition, *only* that multiplication by each single element be a continuous

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