ON SUBSURFACES OF SOME RIEMANN SURFACES

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Introduction. In the theory of meromorphic functions, it is important to investigate the properties of covering surfaces generated by their inverse functions. For this purpose, the study of properties of a non-compact region of a Riemann surface is useful.

Recently Kuramochi has given in his paper [5] the following very interesting theorem. Let R be a Riemann surface and let R_0 be a compact domain on R with compact relative boundary ∂R_0 . Then

Theorem. If R belongs to $O_{HB} - O_G$ ($O_{HD} - O_G$ resp.), then $R - R_0$ belongs to O_{AB} (O_{AD} resp.).

Here we use the following notations.

 O_{g} : the class of Riemann surfaces which admit no Green function.

 $O_{HB}(O_{AB})$: the class of Riemann surfaces on which there exists no nonconstant single-valued bounded harmonic (analytic) function.

 $O_{HD}(O_{AD})$: the class of Riemann surfaces on which there exists no nonconstant single-valued harmonic (analytic) function with finite Dirichlet-integral.

Constantinescu-Cornea [1] have investigated this theorem in detail and obtained several results. Kuramochi [6] has extended this theorem again.

On the other hand, the method given by Heins [2] may be expected to contribute to the same purpose. He introduced the concept "locally of type-Bl" using the Green functions and gave many results concerning covering properties.

We shall give, in this article, simple proofs of extended Kuramochi's theorems in Constantinescu-Cornea's way and prove some properties of covering surfaces using them and Heins' method.

For simplicity, we shall call, in this article, a non-compact or compact domain G on a Riemann surface R a subregion on R when its relative boundary C with respect to R consists of at most an enumerable number of analytic noncompact or compact curves which cluster nowhere in R. We say that G belongs

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