ON THE LOCAL THEORY OF CONTINUOUS INFINITE PSEUDO GROUPS I.*

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Introduction

The local theory of continuous (infinite) pseudo-groups of transformations was originated by S. Lie, and developed by himself, F. Engel, E. Vessiot, E. Cartan, etc. In the beginning, the definition was not clear and we can find several different definitions in the papers of pioneers. In 1902, E. Cartan introduced a definition using his theory of exterior differential systems and made an extensive study in his series of papers [1], [2], and [3]. The writer will adopt his definition in this series of papers. A continuous pseudo-group of transformations is, roughly speaking, a collection of real (or complex) analytic homeomorphisms of domains in a real (or complex) euclidean space, which is closed under the operations of composition and inverse, and which forms the general solutions of a system of partial differential equations. An example is the collection of conformal mappings of domains in a complex plane, considered as a real euclidean space, because the collection forms the general solutions of Cauchy-Riemann equations. A continuous pseudo-group of transformations is called finite, if the underlying system of differential equations is completely integrable, otherwise infinite. Aside from the applications of the theory to the differential geometry and partial differential equations, he was also interested in the analytic-algebraic structure which lies behind the structure of continuous pseudo-group of transformations. Namely, if G is a pseudo-group of transformations and f, g are in G, then the inverse f^{-1} is defined and the composition $f \circ g$ is defined for some pairs (f, g). Thus G forms an algebraic system

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