ON THE VOLUME IN HOMOGENEOUS SPACES

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Guldin-Pappus's theorem about the volume of a solid of rotation in the euclidean space has been generalized in two ways. G. Koenigs [1] and J. Hadamard [2] proved that the volume generated by a 1-parametric motion of a surface D bounded by a closed curve c is equal to $\sum_i a_i b_i + \sum_{i<j} a_{ij} b_{ij}$, where a_i, $a_{ij} = -a_{ji}$ $(i, j = 1, 2, 3)$ are quantities attached to D with respect to a rectangular coordinate system, while b_j, $b_{ij} = -b_{ji}$ $(i, j = 1, 2, 3)$ are quantities determined by our motion. It is remarkable that a_i, a_{ij} depend only on c and not on D. The theorem was extended to the case of dimension n by G. Guillaumin [3]. Another extension of Guldin-Pappus's theorem was obtained by the author [7] in the following way. The volume V of a solid B in the euclidean space of dimension n is given by $\int v d\sigma$, where v is an $(n-1)$-dimensional volume of a section of B by one of the 1-parametric set of hyperplanes and $d\sigma$ is a component, orthogonal to the hyperplanes, of an arcelement of the locus of the center of gravitation of the section. An analogous result was obtained for the spherical space. In the present paper the author generalizes these results to the case of homogeneous spaces by the method of moving frames of E. Cartan and applies the results to various spaces, and states formulas of the integral geometry in the homogeneous spaces.

1. Preliminaries

1. In the first we quote the matters necessary for our purpose from [4] and [6]. Let G be a group which operates on a space M effectively and transitively. We take an element p_0 of M and a set H of all elements of G which fix p_0. Any element p of M corresponds to a set σH $(\sigma \in G)$ of G/H and M can be identified with G/H in natural way. Now we take a set of elements upon which G operates simply transitively and call each element of the set a frame of M, and to a point p corresponds a set of frames σHR, where R is a

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