IMBEDDING INFINITELY DISTRIBUTIVE LATTICES COMPLETELY ISOMORPHICALLY INTO BOOLEAN ALGEBRAS

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Introduction. It is well known that any distributive lattice can be imbedded in a Boolean algebra ([1], [2], [4] and others). This imbedding is in general only finitely isomorphic in the sense that the imbedding preserves finite sums (supremums) and finite products (infimums) (but not necessarily infinite ones). Indeed, in order to be able to be imbedded into a Boolean algebra completely isomorphically (i.e. preserving every supremum and infimum) a distributive lattice L must satisfy the infinite distributive law, as the infinite distributivity holds in Boolean algebras. The main purpose of this paper is to prove that the converse is also true, that is, any infinitely distributive lattice can be imbedded completely isomorphically in a Boolean algebra (Theorem 6). Since we show, on the other hand, that any relatively complemented distributive lattice is infinitely distributive (Theorem 2), Theorem 6 implies that every relatively complemented distributive lattice can be imbedded completely isomorphically in a Boolean algebra (Theorem 4).

The importance of completely isomorphic imbedding is pointed out, as an example, in §5. Thus, theorems on higher degree of distributivity in Boolean algebras are generalized to those in relatively complemented distributive lattices. The theory of higher degree of distributivity in Boolean algebras has been discussed by several authors ([6], [7] and others). To generalize the theory to a relatively complemented distributive lattice L, we first imbed L into a Boolean algebra M completely isomorphically in such a manner that a property P in L will be preserved in M. If the property P implies a property Q in M, then P implies Q in L. By this method we can prove, for example, that any $(\alpha, 2)$-distributive relatively complemented lattice is (α, α)-distributive (Theorem 7).

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