HOMOGENEOUS SPHERE BUNDLES AND THE ISOTROPIC RIEMANN MANIFOLDS

TADASHI NAGANO

Introduction

Let M be a connected metric space and H an isometry group of M which leaves fixed a point p in M. M is said *H-isotropic* at p when, for any two points q and r of M at the same distance from p, there exists an isometry in H which carries q to r. When H coincides with the maximum isometry group leaving p fixed, M is said merely *isotropic* at p.

Assuming further that M is compact and the isometry group of M is transitive, H. C. Wang [26] proved that M is a homogeneous space of a compact Lie group and that the homogeneous space is one of the spaces: a sphere, a (real, complex or quaternion) projective space and the Cayley projective plane.

J. Tits [25] and H. Freudenthal [11] succeeded in determining the homogeneous space M under weaker conditions. Here are Freudenthal's hypotheses on M, which are more general than those of Tits. M is a connected locally compact Hausdorff space. M admits a transitive group I of topological transformations having the properties (S), (V) and (Z).

(S): Given a compact subset A and a closed subset B of M with $A \cap B = \phi$ (the empty set), there exists an open set $U \neq \phi$ such that for any τ in I, $\tau(U) \cap A \neq \phi$ implies $\tau(U) \cap B = \phi$.

By (S), M has an I-invariant uniformity, from which he defines a uniformity on I so that I is a topological group.

(V): I is complete.

(Z): J denoting the isotropy subgroup of I at a point p, there exists an orbit $J(q)$, $q \in M$, such that $M - J(q)$ is not connected.

J is compact. Using Yamabe's theorem, Freudenthal showed that I is a Lie group, and so M admits an I-invariant Riemannian metric. M is then J-isotropic at p. He determined I, J and $M = I/J$ by studying the Lie algebras

Received October 30, 1958.