ON RELATIVE HOMOLOGICAL ALGEBRA OF FROBENIUS EXTENSIONS

KAZUHIKO HIRATA

In this paper we study two features of relative homological algebra of Frobenius extensions, the one is the relative dimension and the other is the relative complete resolution. The theory of Frobenius extensions was developed by F. Kasch [4] as a generalization of Frobenius algebras. We deal with only Frobenius extensions of finite rank (see § 2 for the definition).

Let P be a Frobenius extension ring of its subring Q. In § 3, we prove that the relative global dimension of (P, Q) is 0 or ∞, and if further P is an algebra over a commutative ring, the relative dimension of (P, Q) is 0 or ∞.

For a Frobenius algebra A over a field K, T. Nakayama [5] has constructed a complete resolution for A as a generalization of that for finite groups. On the other hand, a relative complete resolution for (G, H), where G is a group and H is a subgroup of finite index, has been constructed by G, Hochschild [3], and in this case the group ring $Z(G)$ is a Frobenius extension algebra of $Z(H)$. In general the ordinary homology and cohomology of a supplemented K-algebra (K is a commutative ring) is obtained from the Hochschild homology and cohomology groups, if the algebra is K-projective. We show that this holds without any assumption in the relative case (§ 4). In particular, the relative homology of group extensions can be treated in the form of the relative homology of algebras. Now, we construct a relative complete resolution for Frobenius extension algebras (§ 5), as the unification of that for Frobenius algebras and that for group extensions.

Throughout this paper we assume that all rings have a unit element which operates as the identity on all modules, and subrings contain the unit element.

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