A REMARK ON FROBENIUS EXTENSIONS AND ENDOMORPHISM RINGS

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In his paper [1] F. Kasch developed a theory of Frobenius extensions as a generalization of the theory of Frobenius algebras. In it he established a very interesting relationship between the Frobenius property of an extension and that of its endomorphism ring [1, Satz 5], from which he further derived the Frobenius extension property of Galois extensions of simple rings [1, Satz 6]; with these results he kindly responded to what had been vaguely "conjectured" (as he wrote) by one of the writers on the connection between Galois theory and the theory of Frobenius algebras [2]. However, his theory of Frobenius extensions is constructed upon the assumption that the ground ring, A, satisfies the minimum condition (for left ideals and right ideals) and, moreover, the condition that the left annihilator, in A, of a right ideal in A different from A should not vanish and similarly the right annihilator, in A, of a left ideal different from A should not vanish; he calls such a ring an S-ring. His proof to his above alluded theorem on the relationship between Frobenius extensions and endomorphism rings depends also to this assumption, and particularly to the last condition. The purpose of the present note is to free the theorem from this condition (and even from the minimum condition) establishing it for the case of an arbitrary ground ring.

(It is desireable to free also some other parts of the theory from the same S-ring assumption, though for some parts of the theory the assumption is rather natural and perhaps necessary, and the job will be taken up in a subsequent paper, to appear somewhere, in which generalizations of the theory in other contexts will be considered too.)

1. Preliminaries

Let $\mathfrak{S}$ be a ring having a unit element 1, and A be a subring of $\mathfrak{S}$ which contains 1. The module $\mathrm{Hom}_A(\mathfrak{S}_A, A_A)$ of A-right-homomorphisms of $\mathfrak{S}$ into A

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