A FUNCTION ALGEBRA ON RIEMANN SURFACES

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1. Introduction. In this note, we treat the problem to determine the conformal structure of the closed surface by the structure of the differentiable function algebra as the normed algebra with a certain norm.

A similar investigation is found in Myers [1]. He concerns himself with determining the Riemannian structure of the compact manifold using a certain normed algebra of differentiable functions.

We have shown in [2] the fact that the Royden's ring as a topological ring determines the quasiconformal structure of the Riemann surface. Thus it is natural to inquire whether *the Royden's ring as a normed ring characterizes the Riemann surface* or not. This problem is positively answered for closed surfaces by reduction to the following: *A topological mapping between two surfaces with the annular maximal dilatation*[1] *1 is a conformal*[2] *mapping.*

2. Royden's ring. We denote by R an open or closed Riemann surface and by $M(R)$ its Royden's ring, i.e., the normed ring of all bounded continuous functions on R which are absolutely continuous in the sense of Tonelli[3] with finite Dirichlet integrals. The norm of f in $M(R)$ is given by

$$\|f\| = \|f\|_\infty + \sqrt{D[f]}, \tag{1}$$

where $\|f\|_\infty$ denotes the uniform norm $\sup(|f(P)|\ ;\ P \in R)$. Then $M(R)$ is a complete normed ring with respect to the norm (1).

We denote by $C^n \cap M(R)$ the incomplete normed subring of $M(R)$ consisting of all C^n-functions in $M(R)$. The following holds (cf. [2]).

LEMMA 1. *$C^n \cap M(R)$ is dense in $M(R)$ $(n = 1, 2, \ldots)$.*

Received March 16, 1959.

1) The definition will be given in §3.

2) Here and hereafter the term *conformal* includes both of the *direct* and the *indirect* one.

3) A function $f(x, y)$ on $[a, b;\ c, d]$ is called *absolutely continuous in the sense of Tonelli* if $f(x, y)$ is absolutely continuous in $x \in [a, b]$ for almost every fixed values $y \in [c, d]$ and the corresponding fact holds if x and y are interchanged and further f_x and f_y are locally integrable. The notion is carried over Riemann surfaces using local parameters.