

# ON A RING ISOMORPHISM INDUCED BY QUASICONFORMAL MAPPINGS

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## Introduction

The purpose of this paper is to study the relationship between a certain isomorphism of some rings of functions on Riemann surfaces and a quasi-conformal mapping.

It is well known that two compact Hausdorff spaces are topologically equivalent if and only if their rings of continuous functions are isomorphic. We shall establish an analogous result concerning a function ring on a Riemann surface and the quasi-conformal equivalence.

As one of the important properties of quasi-conformal mapping is its absolute continuity in the sense of Tonelli, it is natural to consider the ring of functions which are absolutely continuous in the sense of Tonelli. On the other hand, we can show that this ring, with an additional condition, is coincident with a normed ring considered by Royden [10], which we shall call Royden's ring.

This leads us to study of the correspondence of the ideal boundary defined by using the above normed ring under a quasi-conformal mapping.

Our main results are, roughly speaking, as follows.

*Two Riemann surfaces are quasi-conformally equivalent if and only if their Royden's rings are isomorphic in some sense.*

This can be considered as a ring-theoretic characterization of quasi-conformality.

*A quasi-conformal mapping between two Riemann surfaces can be continuously extended to their "ideal boundaries" in an appropriate manner.*

This includes the invariance of the classes  $O_G$  and  $O_{HD}$  of Riemann surfaces by a quasi-conformal mapping.

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