ON META-ABELIAN FIELDS OF A CERTAIN TYPE

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Let k be an algebraic number field of finite degree, and l a rational prime (including 2); k and l being fixed throughout this paper. For any power l^n of l, denote by ζ_n an arbitrarily fixed primitive l^n-th root of unity, and put $k_n = k(\zeta_n)$. Let r be the maximal rational integer such that $\zeta_r \in k$ i.e. $k_r = k$ and $k_{r+1} \neq k$.

S. Kuroda [7] shows that the decomposition law of rational primes in some absolute non-abelian normal extension is determined by the rational 2^2-th power residue symbol of Dirichlet, to which A. Fröhlich [1] gives a more general apprehension. L. Rédei defined in [8] a new symbol, which he called "bedingtes Artinsches Symbol" (restricted Artin symbol), and he established in [9] a theory concerning Pell's equations by means of this symbol.

In the present paper, we define in § **1** the "restricted l^n-th power residue symbol", which is related to the restricted Artin symbol in the same manner as the ordinary power residue symbol to the ordinary Artin symbol. The restricted l^n-th power residue symbol is a generalization of Dirichlet's symbol mentioned above. So we investigate some meta-abelian extensions over k, for which the decomposition law of prime ideals of k is given by means of the restricted l^n-th power residue symbol. More precisely, let A/k be an abelian extension over k and $\mathfrak{K}/A$ a kummerian extension of A obtained by adjoining to A the l^{n_i}-th roots ω_i of numbers a_i in k $(i=1, \dots, t)$. We call a normal subfied M of $\mathfrak{K}$ a *k-meta-abelian l-field over k*, or simply *k-meta-aeblian*, if M contains all the l^{n_i}-th roots of unity. Then the decomposition law of prime ideals of k in a k-meta-abelian l-field is determined. This result is a generalization of that of Kuroda [7] concerning P-meta-abelian 2-field over P, P being the rational number field.

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