ON RAMIFIED RIEMANN DOMAINS

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Let φ be a holomorphic mapping of an n-dimensional analytic space E into C^n. If φ is non-degenerate at every point of E, we call the pair (E, φ) a Riemann domain. The notion of a Riemann domain is a generalization of the notion of a concrete Riemann surface. A Riemann domain (E, φ) is said to be unramified if φ is a local homeomorphism, and to be ramified if otherwise.

In the classical theory of functions of several complex variables, most considerations have been done only on unramified Riemann domains. If we permit (holomorphic-) algebroidal functional elements in the analytic prolongation, there appears the ramified Riemann domain. Some of the classical results concerning unramified Rimann domains do not hold for ramified Riemann domains, and we know very little to what extent the results in the unramified case can be generalized to the ramified case. The purpose of the present paper is to investigate some properties of ramified Riemann domains.

In §**1** we recall the notion of analytic spaces and introduce some terminologies for the later use. In §**2** a Riemann domain and its boundary are defined, and their properties are stated. After these preparations we consider in §**3** the holomorphic prolongations of holomorphic functions on a Riemann domain. As in the unramified case, for a holomorphic function f on a Riemann domain we can construct canonically the Riemann domain of the maximal holomorphic prolongation of f or the existence domain of f. Thus we arrive at the notion of the *domain of holomorphy*. It is well known that in the unramified case a domain of holomorphy is characterized by its holomorphic convexity. However in general cases such characterization is impossible.[1] So we are led to consider a certain condition which is strictly weaker than the holomorphic convexity, and to prove its sufficiency for a Riemann domain to be a domain of holomorphy; this forms a main result of the present paper and is given in §**4** Theorem 4.

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[1] Cf. H. Grauert and R. Remmert [8].