HOLOMORPHIC FUNCTIONS WITH SPIRAL ASYMPTOTIC PATHS

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1. Let $f(z)$ be a holomorphic and unbounded function in $|z| < 1$, with the property that it remains bounded on some spiral S in $|z| < 1$ which approaches $|z| = 1$ asymptotically. The existence of such functions was first established by G. Valiron.[1] Accordingly, we shall refer to such functions as *functions of class* (V) *relative to* S. More recently, F. Bagemihl and W. Seidel obtained examples of functions holomorphic and unbounded in $|z| < 1$ which approach prescribed finite or infinite values as $|z| \to 1$ on any given enumerable set of disjunct spirals which approach $|z| = 1$ asymptotically,[2] as well as on certain sets of such spirals having the power of the continuum.[3]

In his 1936 paper, Valiron established various properties of functions of class (V) relative to a spiral S, of which we mention, for future reference, the following:

I. *If $f(z)$ is of class (V) relative to a spiral S, there exists a spiral path on which $f(z)$ tends to infinity.*

In this paper, we continue the study of functions with spiral asymptotic paths, and shall derive some further properties of such functions.

2. In the sequel, we shall use the term "spiral" in the following sense. Let $\zeta(t)$ be a continuous, complex-valued function for $0 \leqq t < \infty$ with the properties:

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[1] G. Valiron. 1. *Sur certaines singularites des fonctions holomorphes dans un cercle*, Comptes Rendus de l'Académie des Sciences de Paris, vol. **198** (1934), pp. 2065-2067, and 2. *Sur les singularités de certaines fonctions holomorphes et de leurs inverses*, Journal de Mathématiques pures et appliquées (9), vol. **15** (1936), pp. 423-435.

[2] F. Bagemihl and W. Seidel. *Spiral and other asymptotic paths, and paths of complete indetermination, of analytic and meromorphic functions*, Proceedings of the National Academy of Sciences, vol. **39** (1953), pp. 1251-1258.

[3] F. Bagemihl and W. Seidel. *Some boundary properties of analytic functions*, Mathematische Zeitschrift, vol. **61** (1954), pp. 186-199.