AN INVESTIGATION ON THE LOGICAL STRUCTURE OF MATHEMATICS (VIII)*

CONSISTENCY OF THE NATURAL-NUMBER THEORY $T_1(N)$

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Preliminaries and Consequences

1. Consistency proof and intuitive knowledge

The consistency of the natural number theory was proved, as is well known, by G. Gentzen in 1935 for the first time in such generality that the mathematical induction can be consistently used for any arbitrary predicate of natural numbers, which is well-formed in his system so that every quantifier ranges over all natural numbers. His formulation[1] of the natural number theory will be for simplicity referred to as GN. In GN the natural numbers 0, 1, 2, . . . are represented by special symbols of the system GN. Some predicate of natural numbers, such as $*<*$, $*=*$, etc., and some operations, such as $*+*$, $*\times*$, etc., are also represented by special symbols of GN. These special symbols of GN must be such that their intuitive interpretations are allowed in such a way that the intuitive truth and falsehood of those statements which are construed by variables for natural numbers, the special symbols, and the connectives of propositional logic, without using quantifiers, can be determined by our intuitive knowledge. Conversely, any predicates and operations which have this property can be used as special symbols of GN. From among the statements of the above mentioned form, the "mathematischen Grundsequenzen", such as $\rightarrow a=a$, $\rightarrow a<b \wedge b<c \rightarrow a<c$, etc. are extracted as such statements that are intuitively true, i.e. true for any arbitrary substitution of natural numbers for all the variables occurring in the statements. This is the basis for the fact that the formal system GN is related to our intuition of natural numbers. For this reason

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* This Part (VIII) depends logically only on Part (I), §1–§11 and Part (II), §12–§15, Hamburger Abh. vol. 22. The superscript such as §11, (I) is the reference to §11, Part (I).

[1] In the sequel we refer to his second formulation given in Semesterber. Münster (1938).