AN INVESTIGATION ON THE LOGICAL STRUCTURE OF MATHEMATICS (VI)[0)]

CONSISTENT V-SYSTEM T(V) (WITH CORRECTIONS TO PART (XII))

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The V-system T(V) is defined in §2 by using §1, and its consistency is proved in §3. The definition of T(V) is given in such a way that the consistency proof of T(V) in §3 shows a typical way to prove the consistency of some subsystems of UL. Otherwise we could define T(V) more simply by using truth values. After T(V)-sets are treated in §4, it is proved in §5 as a T(V)-theorem that T(V)-sets are all equal to V. In this proof a peculiar T(V)-set $\tilde{R}$, defined similarly as Russell's contradictory set R, is used. The V-system itself is a trivial subsystem of UL, while such a subsystem S of T(V) is important, for which S-unprovability of $\forall x.\ x = V$ is known (§6). As an example for such a subsystem of T(V), the consistent natural number theory $T_1(\tilde{N}) \cap T(V)$ is developed in §7.[0)] The sequence V, {V}, {{V}}, ... is the sequence of natural numbers in $T_1(\tilde{N})$. In §8 the dual 0-system T(0) is defined. As a consequence, the most general duality principle in logic is obtained. In §9 the method of making use of the V-system in proving the consistency of some part of mathematics is briefly described. As an appendix a method to construct an infinitely many number of inconsistent systems of dependent variables is added.

1. Decomposition of a formula in UL

Let G be any formula in UL. The trees $T_0, T_1, \ldots$ are constructed successively from G as follows. T_0 is the tree consisting only of G. Assume that $T_k (k \geq 0)$ has been constructed, and that there is a bottom formula, say

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[0)] Continuation of the author's previous work with the same major title. This Part (VI) presupposes in particular the terminologies and the knowledge in Parts (I) and (II), forthcoming in Hamburger Abhandlungen, and §§7, 9 of this Part (VI) can only be understandable after reading §§7, 10, 11 of Part (VII), appearing in this volume. The definition of concept and set is given in Part (X) forthcoming elsewhere. The references indicated by upper suffixes (I), (II), ... refer to the Part (I), (II), ... respectively.