ON REAL IRREDUCIBLE REPRESENTATIONS OF LIE ALGEBRAS

NAGAYOSHI IWAHORI

§ 1. Introduction

Let us consider the following two problems:

Problem A. *Let* $\mathfrak{g}$ *be a given Lie algebra over the real number field* R. *Then find all real, irreducible representations of* $\mathfrak{g}$.

Problem B. *Let* n *be a given positive integer. Then find all irreducible subalgebras of the Lie algebra* $\mathfrak{gl}(n, R)$ *of all real matrices of degree* n.

In a beautiful and fundamental paper [1], E. Cartan solved completely the Problem B, in the sense that he gave a method to determine all the subalgebras of $\mathfrak{gl}(n, R)$ by a finite process, and determined them actually for the case $n \leqq 12$ for which he gave a table. As we shall see in § 6, 7, the Problem A is reduced to the one to find all complex irreducible representations and to distinguish among them those representations which are of the first class, and then the Problem A is easily reduced to the reductive case, i.e. to the case where $\mathfrak{g}$ is reductive. As a reductive Lie algebra is a direct sum of simple Lie algebras, the Problem A can be further reduced to the case where $\mathfrak{g}$ is simple, as we shall see later. Now if the Problem A could be solved for every Lie algebra $\mathfrak{g}$, then one has only to look at the table to solve B. In analysing [1] closely, we notice that E. Cartan solved the Problem B by this principle. In several places of [1], E. Cartan has recourse to verifications for each type of simple Lie algebras A, B, C, D and the results of verifications for exceptional cases are stated without proof.

In the present paper, we shall solve the Problem A by the above mentioned principle and reestablish the results of [1]. The knowledge of [1] is not presupposed for the reader. Where E. Cartan had recourse to verifications for each type of simple algebras, we shall be able to obtain the corresponding results by general considerations.

Received May 31, 1958.