SOME REMARKS ON NON-COMMUTATIVE EXTENSIONS OF LOCAL RINGS

EDWARD H. BATHO

In [1]* we introduced the concept of a non-commutative local ring and studied the structure of such rings. Unfortunately, we were not able to show that the completion of a local ring was a semi-local ring. In this paper we propose to study a class of rings for which the above result is valid. This class of rings is the integral extensions-[4, 5]-of commutative local rings. This class of rings includes the important class of matrix rings over commutative local rings. In part 1 below we study some elementary properties of integral extensions and here we assume merely that the underlying ring is semi-local. In part 2 we discuss some questions of ideal theory for arbitrary local rings as well as for integral extensions. In a later paper we propose to utilize our results to study the deeper properties of these rings including a dimension theory for such rings. We are particularly indebted to the work of Nagata [6, 7, 8, 9] in the preparation of this paper.

Part 1. Elementary properties

If R is a ring, $J(R)$ will designate the Jacobson radical of R. Throughout "ideal" will mean two-sided ideal.

(1.1) Def: If R is a ring with identity, R is said to be semi-local if:

(a) R satisfies the maximum condition for left ideals;

(b) $R/J(R)$ satisfies the minimum condition for left ideals;

(c) $\bigcap_{n=0}^{\infty} J^n(R) = 0$, $J^0(R) = R$.

If, in addition, R satisfies:

(d) $J(R)$ is a unique maximal ideal in R,

then R is said to be a local ring. See [1].

Received June 3, 1958.
Revised July 15, 1958.

* See Bibliography at end of paper.