A REMARK ON THE COMMUTATIVITY OF ALGEBRAIC RINGS

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Let F be a commutative ring with unit element 1. A ring R is called an algebraic ring over F by Drazin, in his recent paper [2], when R is an algebra[1] over F and every element x of R satisfies an equation of *lower monic* form

$$x^m + \alpha_{m+1}x^{m+1} + \ldots + \alpha_n x^n = 0$$

with coefficients α_i in F. With this terminology and in generalization of some results of Arens-Kaplansky [1], Herstein [5], Jacobson [6], he proves in the same paper:

Let F be either i) *the ring of rational integers*, or ii) *a field of prime characteristic algebraic over its prime field*, or iii) *an algebraically closed field*. Then, *if R is an algebraic ring over F and if every nilpotent element in R is central, R is commutative.*

Now, a common feature in i) and ii) is that F is generated by a single element. Indeed, by making use of the writer's [9] generalization of Herstein's [5] theorem we may prove that the above statement holds whenever F is (ring-)generated by a finite number of elements.[2] Also the case iii) can correspondingly be generalized to the case of a (commutative) ring which is (ring-)generated by an algebraically closed subfield K and a finite (possibly void) set of elements. A further generalization is obtained by replacing here the algebraically closed subfield K with a subring K such that an integrity domain which is a homomorphic image of K is always an algebraically closed field. We may also regard the case with a finite set of generators as having

Received July 18, 1958.

[1] Thus R admits F both as a ring of left operators and a ring of right operators and the equations $\alpha x = x\alpha$, $\alpha(xy) = (\alpha x)y$, $(xy)\alpha = x(y\alpha)$ hold for all $x, y \in R$ and $\alpha \in F$. Together with Drazin [2] we explicitly remark that a non-zero element of F may induce 0-endomorphism on R.

[2] A direct application of [9], Satz 2 is allowed provided R possesses a unit element. The case of R having no unit element can, however, be settled by an easy modification of the proof there; cf. below too.