TRANSFORMATION GROUPS WITH $(n-1)$-DIMENSIONAL ORBITS ON NON-COMPACT MANIFOLDS

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Introduction

When a Lie group G operates on a differentiable manifold M as a Lie transformation group, the *orbit of a point p in M under G*, or the *G-orbit* of p, is by definition the submanifold $G(p) = \{G(p)\,;\ g \in G\}$. The purpose of this paper is to characterize the structure of a non-compact manifold M such that there exists a compact orbit of dimension $(n-1)$, $n = \dim M$, under a connected Lie transformation group G, which is assumed to be compact or an isometry group of a Riemannian metric on M. When G is compact there exists on M a G-invariant Riemannian metric, and so we shall always consider G as an isometry group. In order to state our main theorem we need another definition: a Riemann manifold M is said *isotropic* (or *H-isotropic*) at a point p in M when there exists an isometry group H of M such that, for any two unit vectors X and Y at p, H contains an isometry carrying X to Y (Some authors use this terminology in a different sense). Now the main theorem (Theorem 3) reads: If there exists a compact $(n-1)$-dimensional G-orbit then M admits a fibre bundle structure over a compact orbit $B = G(b)$, $b \in B$, associated with the principal bundle $(G, G/H, H)$ where H is the isotropy subgroup at b, the fibre being diffeomorphic to the euclidean space on which the structure group H operates as a linear group. The fibre is a submanifold of M containing b and H-isotropic at b, if $\dim B > n-1$. The hypothesis of the theorem can be replaced by a more geometric one: G leaves invariant and operates transitively on a connected component of any submanifold consisting of the points at a constant distance from a fixed compact submanifold.

P. S. Mostert proved a similiar theorem [5] in a different formulation (see

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