NOTE ON COMPLETE COHOMOLOGY OF A QUASI-FROBENIUS ALGEBRA

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Let A be a quasi-Frobenius algebra over a field K. A has a complete (co)homology theory which may be established upon an augmented acyclic projective complex, i.e. a commutative diagram

(1) $$\begin{array}{c} \cdots \to X_1 \to X_0 \to X_{-1} \to X_{-2} \to \cdots \\ \varepsilon\searrow \nearrow\iota \\ A \end{array}$$

of A-double-modules with exact horizontal row, projective X_p, and with epimorphic resp. monomorphic ε and ι. Negative-dimensional cohomology groups, over an A-double-module, are expected to be in close relationship with (ordinary positive-dimensional) homology groups. Indeed, in case A is a Frobenius algebra the cohomology groups $H^{-n}(A, M)$, $-n < -1$, over an A-double-module M may be identified, connecting homomorphisms taken into account, with the homology groups $H_{n-1}(A, M^*)$ over an A-double-module $M^* = (M, *)$ obtained from M by modifying its A-right-module structure with an automorphism $*$ of A belonging to the Frobenius algebra structure of A, and, moreover, the cohomology groups $H^0(A, M)$, $H^{-1}(A, M)$ are described explicitly in terms of commutation and norm-map, so to speak, defined by a certain pair of dual bases of A. In the present note we want to give the corresponding description of the 0- and negative-dimensional cohomology groups of a quasi-Frobenius algebra A. In doing so, we shall deal with a certain A-double-module $M^{\S}$ which is obtained from M by a certain construction but which is in general not A-left-isomorphic to M contrary to that M^* in case of a Frobenius algebra is A-left-isomorphic to M. Further, our construction will strongly rely upon the relationship of A with its core algebra A_0 which is a Frobenius algebra. In fact, the (co)homology theory of an algebra can, generally, be reduced to that of its core algebra, and this principle applies also to the complete (co)homology of a quasi-Frobenius algebra. However, description and construction in terms

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