FIXED POINTS OF ISOMETRIES

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1. Statement of Theorem

The purpose of this paper is to prove the following

THEOREM. *Let M be a Riemannian manifold of dimension n and let ξ be a Killing vector field* (*i.e., infinitesimal isometry*) *of M. Let F be the set of points x of M where ξ vanishes and let $F = \cup V_i$, where the V_i's are the connected components of F. Then* (*assuming F to be non-empty*)

(1) *Each V_i is a totally geodesic closed submanifold* (*without singularities*) *of M and the co-dimension of V_i* (*i.e.*, $\dim M - \dim V_i$) *is even.*

(2) *The structure group of the normal bundle over V_i can be reduced to $GL(r, C)$, where $2r$ is the co-dimension of V_i.*

(3) *If $x \in V_i$ and $y \in V_j$ and $i \neq j$, then there is a 1-parameter family of geodesics joining x and y provided M is complete; hence x and y are conjugate to each other.*

(4) *If M is, moreover, compact, then the Euler number of M is the sum of Euler numbers of V_i's:*

$$\chi(M) = \Sigma \chi(V_i),$$

(*the summation is well defined, as the number of connected components V_i is finite*).

Remarks. (2) implies that if M is orientable, then V_i is orientable.

If F consists of only isolated points, then (4) is a particular case of the Index Theorem, as the index of a Killing vector field at an isolated zero point is 1.

COROLLARY 1. *Let L be an abelian Lie algebra of Killing vector fields of M. Let F be the set of points x of M where every element of L vanishes. Then the same statements as in Theorem hold.*

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