ON INVARIANT CONNECTIONS OVER A PRINCIPAL FIBRE BUNDLE*

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1. Introduction

The invariant affine connection over a coset space G/J of a Lie group G have been discussed by various authors. Recently, Nomizu [8] gave a systematic study of this problem when J is reductible in G. Among other results, he established a 1-1 correspondence between the invariant affine connections and certain multilinear mappings, and calculated the torsion and curvature. For canonical affine connection of the second kind, the holonomy group was also given.

It is the purpose of this paper to discuss the connections over a principal fibre bundle which admit a fibre transitive[1)] Lie group of automorphisms without restricting to the reductible case. In fact, let $\{E, S\}$ be a differentiable principal fibre bundle with total space E, structural group S, and base space B. Suppose G to be a Lie group of automorphisms of $\{E, S\}$, and J the subgroup leaving a fibre F_0 invariant. There is a natural homomorphism $\psi : J \rightarrow S$. If we regard G as a transformation group of the base space B, then J is the isotropic subgroup at the point $b_0 \in B$ which corresponds to F_0. (In the particular case that E is the bundle of frames of B, then ψ is nothing but the linear representation of J on the tangent space of B at b_0.) Let us denote by $\hat{\mathrm{G}}$, $\hat{\mathrm{S}}$, $\hat{\mathrm{J}}$ the Lie algebras of G, S, J respectively. The main results can be stated as follows:

(A) *Suppose G to be transitive on the fibres of E. Then there is a* 1-1 *correspondence between the G-invariant connections over E and the linear mappings* $\Psi : \hat{\mathrm{G}} \rightarrow \hat{\mathrm{S}}$ *such that* (1) $\Psi \circ \mathrm{Ad}. j = \mathrm{Ad}. \psi(j) \circ \Psi$, $j \in J$, *and* (2) $\Psi(\vec{j}) = \psi(\vec{j})$, $\vec{j} \in \hat{\mathrm{J}}$ *where we use* ψ *to denote both the group homomorphism:* $J \rightarrow S$ *mentioned above and the Lie algebra homomorphism:* $\hat{\mathrm{J}} \rightarrow \hat{\mathrm{S}}$ *it induces.*

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1) By fibre transitive, we mean that, given any two fibres, there exists an element of the group carrying one to the other.