The main purpose of the present paper is to establish a theorem concerning the relation between the group of all projective transformations on an affinely connected manifold and the group of all affine transformations.

We shall say that an affine connection satisfies condition \((E)\), if it is without torsion and affinely complete and if the Ricci tensor field \(S(X, Y)\) is parallel. Our theorem states that if an affine connection satisfies condition \((E)\) and if the quadratic form \(S(X, X)\) is zero or not negative semi-definite, then the two groups coincide. This is just a generalization of the case of ordinary affine space which is well known in analytic geometry.

The proof is based on the theory of normal projective connection introduced by Elie Cartan; in particular, we make use of the "developing" process of this connection. After some preliminaries, in which we follow the book of K. Nomizu [5] for affine connections, we first formulate the normal projective connection from a global point of view, as we shall see in Proposition 1. In § 6, we prove an important lemma (Lemma 8) by using the results in the previous sections (Proposition 1, 2 and 3), from which the main theorem follows immediately. In Appendix, we shall prove a known fact on the geometric characterization of the projective equivalence of two affine connections.

Finally we add that we have obtained some results about the relation between the group of all conformal transformations on a Riemannian manifold and the group of all isometries by using the method analogous to the projective case. We hope to deal with this problem in another paper.

The author expresses his sincere thanks to Professor K. Nomizu for his constant encouragement and interest in this work.

1. Definition of a projective transformation on an affinely connected manifold

Let \(M\) be a connected manifold of class \(C^\infty\). We assume that the dimension