ON AUTOMORPHISMS OF A KÄHLERIAN STRUCTURE

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Is every isometry, or more generally, every affine transformation of a Kählerian manifold a complex analytic transformation? The answer is certainly negative in the case of a complex Euclidean space. This question has been recently studied by Lichnerowicz [8] and Schouten-Yano [11] from the infinitesimal point of view; they have found some conditions in order that every infinitesimal motion of a Kählerian manifold preserve the complex structure. (As a matter of fact, [11] has dealt with the case of a pseudo-Kählerian manifold, which does not differ essentially from a Kählerian manifold as far as the question at hand is concerned.)

In the present paper, we generalize their results by a different approach. In order to explain our main idea, we shall first give a few definitions (**1** and **2**) and state our main results (**3**). The proofs are given in the subsequent sections.

1. Kählerian structures

Let M be a complex analytic manifold of complex dimension n. Its complex structure is defined by a real analytic tensor field I of type $(1, 1)$ with $I^2 = -\mathbf{1}$[1] on the underlying $2n$-dimensional real analytic manifold which satisfies the condition of integrability $I[X, Y] - [IX, Y] - [X, IY] - I[IX, IY] = 0$ for all real vector fields X and Y (for example, [1]). A differentiable transformation f of M is said to preserve the complex structure I if $\delta f \circ I = I \circ \delta f$, where δf denotes the differential of f. This is equivalent to saying that f is a complex analytic transformation. If $\delta f \circ I = -I \circ \delta f$, we say that f maps I into the conjugate complex structure $-I$; f is then a conjugate analytic transformation.

A real analytic Riemannian metric g on a complex analytic manifold M is called Kählerian if it is hermitian, that is, $g(IX, IY) = g(X, Y)$ for all real vector fields X and Y, and if I is a parallel tensor field with respect to the

Received July 23, 1956.

[1] Throughout the present note, $\mathbf{1}$ denotes the identity transformation.