# CANONICAL CONNECTIONS AND PONTRJAGIN CLASSES

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In the previous paper [7], we have studied the relationship between the Riemannian connection of an $n$-dimensional Riemannian space $M$ imbedded into the $(n+k)$-dimensional Euclidean space $R^{n+k}$ and the canonical connection in the bundle $P_{n,k} = O(n+k)/\{1\} \times O(k)$ over the Grassmann manifold $M_{n,k} = O(n+k)/O(n) \times O(k)$.

In the first half of the present paper, the relationship between the canonical connections in bundles $P_{n,k}$, $P^*_{n,k} = O(n+k)/O(n) \times \{1\}$, $O(n)$ over $M_{n,k}$ and the invariant Riemannian connection on $M_{n,k}$ will be discussed. We obtain the holonomy groups of these canonical connections.

In the second half of the paper, we shall study the Pontrjagin classes of manifolds, using exclusively differential forms. To facilitate the calculation, we introduce two types of characteristic cocycles which are closely related to the Pontrjagin cocycles. The duality theorem for the Pontrjagin classes, which has been proved by Wu Wen-Tsun using the cellular subdivision of the Grassmann manifold $M_{n,k}$ [13], [4], [5], is proved here very easily using the theory of symmetric functions. Our result gives a little bit more precise informations than that of Wu Wen-Tsun, in the sense that we express the duality theorem as a relation between the Pontrjagin *cocycles* (in stead of *classes*) and the normal Pontrjagin *cocycles*. This may not be interesting for topologists, but may have some value from the differential geometrical point of view. We prove also that the normal Pontrjagin *cocycles* of a Riemannian space depend only on the Riemannian connection, not on the way how to imbed it. Finally we show that the normal Pontrjagin *classes* of a manifold $M$ depend only on the differentiable structure of $M$.

The second half of the paper (§4–§8) can be read independently of the

Received January 9, 1956; revised July 24, 1956.
* Supported by National Science Foundation Grant.