SOME STUDIES ON KAEHLERIAN HOMOGENEOUS SPACES

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The present paper is devoted to the study of differential geometry of Kaehlerian homogeneous spaces. In section 1 we deal with the canonical decomposition of a simply connected complete Kaehlerian space and that of its largest connected group of automorphisms. We know that a simply connected complete Riemannian space V is the product of Riemannian spaces $V_0, V_1, \ldots, V_n$, where V_0 is a Euclidean space and $V_1, \ldots, V_n$ are not locally flat and their homogeneous holonomy groups are irreducible [2]. Moreover, if V is homogeneous, so are all V_k [10]. We shall show that if V is Kaehlerian space with real analytic metric (resp. Kaehlerian homogeneous space), each factor V_k is also Kaehlerian (resp. Kaehlerian homogeneous) and that V is the product of $V_0, V_1, \ldots, V_n$ as Kaehlerian space. We call this decomposition the de Rham decomposition of the Kaehlerian space V. Although this result is supposedly known, there is no published proof as yet. Using this decomposition theorem we shall show that the largest connected group of automorphisms of a simply connected complete Kaehlerian space with real analytic metric is the direct product of those of the factors of the de Rham decomposition. In the Riemannian case this result has be been established in [3] by one of the authors of the present paper.

On the other hand, a Kaehlerian homogeneous space G/B of a reductive Lie group G is the direct product of Kaehlerian homogeneous spaces $W_0, W_1, \ldots, W_m$, where W_0 is the center of G with an invariant Kaehlerian structure and where $W_1, W_2, \ldots, W_m$ are simply connected Kaehlerian homogeneous spaces of simple Lie groups ([1], [7], [8]). In section 2 we shall show that this decomposition of G/B is equal to the de Rham decomposition of G/B. We shall prove in fact a theorem that the homogeneous holonomy group of a Kaehlerian homogeneous space of a simple Lie group is irreducible. To prove

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