ON THE TRANSITION PROBABILITY OF A RENEWAL PROCESS

TAKEYUKI HIDA

J. L. Doob, D. Blackwell, W. Feller and other authors have obtained several results concerning the renewal theorem. Especially Doob [1] has considered the renewal process and has showed that it becomes a stationary Markov process if we add a certain initial random variable to it. In the present note, we shall study this stationary Markov process and try to determine its transition probability by virtue of a pair of partial differential equations.

The author would like to express his hearty thanks to prof. A. Amakusa who has encouraged him with kind discussions throughout the course of preparing the present note.

§ 1. Preliminary notions

Most of the results in this section will be obtained by referring to the Doob's paper [1].

Let $X_0(\omega)$[1], $X_2(\omega)$, $X_3(\omega)$, . . . be mutually independent non-negative random variables with the common distribution function $G(x)$ such that

$$(1)\qquad P(X_i \leqq x) = G(x) = \begin{cases} \int_0^x g(t)dt & , \text{ if } x \geqq 0, \\ 0 & , \text{ if } x < 0, \end{cases}$$

$$i = 0, 2, 3, \ldots .$$

Furthermore we assume that $g(x) \geqq 0$ belongs to C^1-class and X_i has finite mean and variance.

In the renewal theory, these $\{X_i\}$ ($i = 0, 2, 3, \ldots$) denote the lifetimes of individuals born successively and especially X_0 denotes the lifetime of the one which survives at $t = 0$.

Let x (for which $G(x) < 1$) be the age of it at $t = 0$. Then $X_1 = X_0 - x$ is

Received February 27, 1956.

1) ω is the probability parameter. We shall omit it unless we need it specially.