HOLONOMY GROUPS OF HYPERSURFACES

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The restricted homogeneous holonomy group of an n-dimensional Riemannian manifold is a connected closed subgroup of the proper orthogonal group $SO(n)$ [1]. In this note we shall prove that the restricted homogeneous holonomy group of an n-dimensional *compact* hypersurface in the Euclidean space is actually the proper orthogonal group $SO(n)$ itself. This gives a necessary (of course, not sufficient) condition for the imbedding of an n-dimensional compact Riemannian manifold into the $(n+1)$-dimensional Euclidean space.

The method used here shows how the theory of connections in fibre bundles is efficacious for problems in classical differential geometry. In fact, the essential part in this paper is the notion of *induced cnnnection*, which is outside of the frame of classical differential geometry.

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1. Holonomy groups

Let P be a principal fibre bundle over a manifold M with Lie structure group G and with projection π. Let ω be a g-valued linear differential form on P defining an infinitesimal connection in P [3], where g is the Lie algebra of G. Let u_0 be a point in P. If c is a closed curve in M starting from $x_0=\pi(u_0)$, then the point $c(u_0)$ obtained by the parallel displacement of u_0 along the curve c is in the same fibre as u_0; hence there is a unique element s in G which maps u_0 to $c(u_0)$:

$$c(u_0)=u_0 s.$$

(The structure group G acts on P on the right, as P is a principal fibre bundle.) The set of all elements s obtained in this way forms a subgroup (not necessarily closed) of G, which is called the *holonomy* group of the connection ω with refer-

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