CORRECTIONS TO MY PAPER "ON KRULL'S CONJECTURE CONCERNING VALUATION RINGS"

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The proof of Theorem 1 in the paper "On Krull's conjecture concerning valuation rings" (vol. 4 (1952) of this journal) is not correct.[1] We want to give here a corrected proof of the theorem: From p. 30, *l.* 14 to p. 31, *l.* 7 should be changed as follows.

Further we observe that if $w(a-b)>2\alpha$, then $(x+a)/(x+b)$ is unit in $\mathfrak{o}$. Hence we may assume that $w(a_i-b_j)<2\alpha$ for any (i, j).

Next, we will show two lemmas concerning the valuations w_λ and w_e:

LEMMA A. *Set* $d=\prod_1^{n'}(x+a_i)/\prod_1^{m'}(x+b_j)$ *and assume that* $w(a_i)=w(b_j)=\sigma$ $(a<\sigma<2\alpha)$ *for any* i *and* j. *Let* e *be any element of* K *such that* $w(e)=\sigma$. *Then either* $w_e(d)\geqq w_\sigma(d)$ *or there exists one* b_j *such that* $w_e(d)\geqq w_{b_j}(d)$.

Proof. We may use the induction argument on $m'+n'$. Obviously $w_e(x+a_i)=\min(w(a_i-e), 2\alpha)$, $w_e(x+b_j)=\min(w(b_j-e), 2\alpha)$: Let σ' be the maximum of these values. We renumber a_i and b_j so that $w_e(x+a_i)=w_e(x+b_j)=\sigma'$ if and only if $i\leqq r$, $j\leqq s$. Now it must be observed that $w_e(x+a_i)=w(a_j-a_1)$ or $w(a_i-b_1)$ for $i>r$, according to $r\neq 0$ or $s\neq 0$, and that similar fact holds for b_j.

1) When $r=n'$, $s=m'$ and $r\geqq s$, we have obviously $w_e(d)\geqq w_\sigma(d)$.

2) When $r<s$ and $r+s\neq m'+n'$: Set $d'=\prod_1^r(x+a_i)/\prod_1^s(x+b_j)$. Then $w_e(d')>w_\sigma(d')$ and therefore there exists on b_j $(j\leqq s)$ such that $w_e(d')\geqq w_{b_j}(d')$. Since the values of factors of d other than those of d' are invariant under the replacement of w_e by w_{b_j}, we have $w_e(d)\geqq w_{b_j}(d)$.

3) When $r=n'$, $s=m'$ and $r<s$: Let $\sigma*$ be the minimum of values $w(a_i-a_{i'})$, $w(a_i-b_j)$ and $w(b_j-b_{j'})$ and let $e*$ be an element of K such that $w(a_i-e*)=w(b_j-e*)=\sigma*$ for any i, j.[2] Then since $w_e(d)\geqq w_{e*}(d)$, we

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1) Prof. P. Ribenboim has communicated to the writer that the proof is not correct. The writer is grateful to him for his kind communication.

2) Such elements e^*, e'' and so on exist because K is algebraically closed and therefore the residue class field of the valuation ring of w is algebraically closed (and contains infinitely many elements).